Sorting Algorithms

Data Structures and Algorithms Tutorial

IIITH - S26

Bubble, Selection, Insertion, Merge, Quick Sort

Agenda

1. Overview
2. Simple Sorts
3. Merge Sort
4. Quick Sort
5. Practice Problems

Overview

What is Sorting?

Definition

- Arranging elements in a specific order (ascending/descending)

Simple Sorts

Bubble Sort

Main Idea

- Repeatedly swap adjacent elements if they're in wrong order
- $O(n^2)$ time, $O(1)$ space

Algorithm

1. Compare adjacent elements from left to right
2. Swap if left element > right element
3. One pass = largest element at correct position
4. Repeat until no swaps needed

Insight

- After i passes, last i elements are sorted
- Can early exit if no swaps in a pass (already sorted!)
- Best case: $O(n)$ when already sorted

Selection Sort

Main Idea

- Find minimum element in unsorted portion -> Place it at front
- $O(n^2)$ time, $O(1)$ space

Algorithm

1. Find minimum in unsorted portion (index i to $n - 1$)
2. Swap it with element at position i
3. Move boundary: i++
4. Repeat until sorted

Insight

- Minimal swaps: Only $n - 1$ swaps total (great for expensive swaps!)
- Not adaptive: Always $O(n^2)$ even for sorted arrays
- Good when: Write operations are expensive (e.g., flash memory)

Insertion Sort

Main Idea

- Build sorted array one element at a time
- $O(n^2)$ time, $O(1)$ space

Algorithm

1. Take next element from unsorted portion
2. Insert it into correct position in sorted portion
3. Shift elements to make space

Insight

- Adaptive: Fast for nearly sorted data ($O(n)$ best case!)
- Stable: Maintains relative order of equal elements
- Online: Can sort as it receives data (Timsort uses it!)

Merge Sort

Merge Sort: Intuition

How would you sort LOTS of numbers?

- Don't tackle it all at once
- Split into smaller problems -> Solve each small problem
- Combine solutions!

Analogy ig?

- Two students each have sorted piles of exam papers
- To merge: Look at top of both piles, pick smaller one
- Repeat until both piles empty

Insight

- Merging two sorted lists takes $O(n)$ time
- If we split recursively, we get $O(n \log n)$ total!

Merge Sort: Visualization

Merge Sort Viz

Merge Sort: Why O(n log n)?

Recurrence Relation: $T(n) = 2T\left(\frac{n}{2}\right) + O(n)$

- Split into 2 halves: $2T\left(\frac{n}{2}\right)$
- Merge takes linear time: $O(n)$

Tree Method

- Level 0: 1 problem of size $n \longrightarrow$ cost: n
- Level 1: 2 problems of size $\frac{n}{2} \longrightarrow$ cost: n
- Level 2: 4 problems of size $\frac{n}{4} \longrightarrow$ cost: n
- …
- Level $\log n$: n problems of size 1 $\longrightarrow$ cost: n

Total: $n \times \log n$ levels = $O(n \log n)$

Space complexity: $O(n)$ for temporary arrays during merge

Merge Sort: Pros and Cons

Advantages

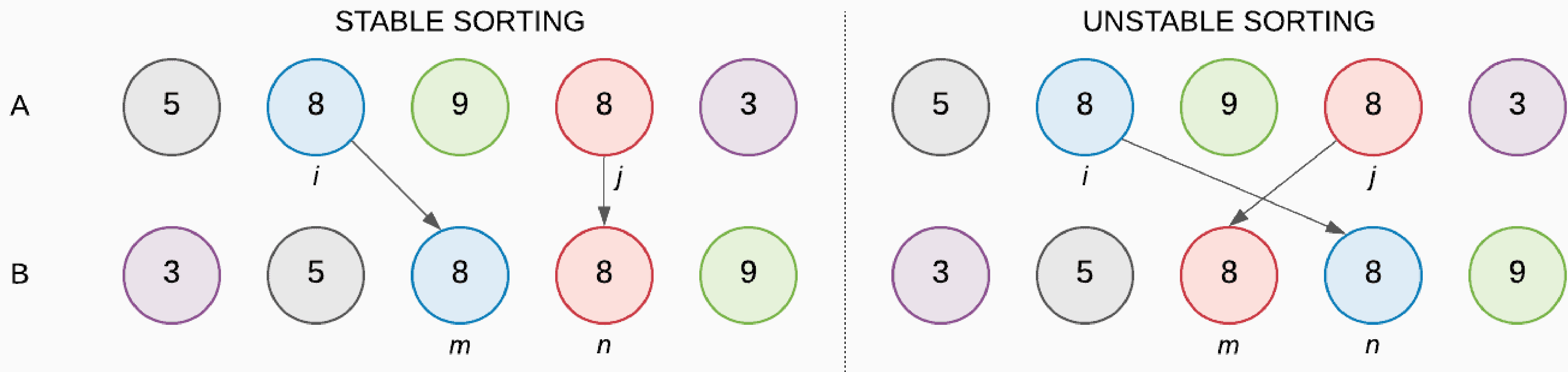
1. Guaranteed performance: Always $O(n \log n)$, no worst case!
2. Stable sort: Maintains order of equal elements
3. External sorting: Works well with files too large for memory (Here memory refers to RAM)
4. Parallelizable: Left and right halves can be sorted independently

Disadvantages

- Space complexity: Requires $O(n)$ extra space, basically not in-place!
- Slower in practice: Quick Sort often faster due to cache locality

When to use?

- When guaranteed $O(n \log n)$ is critical
- When stability is required


STABLE SORTING
UNSTABLE SORTING
A
B
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m
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i
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m
n

Quick Sort

Quick Sort: Intuition

Insight

Don’t worry about sorting everything perfectly immediately. Just:

- Pick one element as “pivot”
- Put everything smaller on left
- Put everything larger on right
- Pivot is now in correct final position!
- Recursively sort left and right

Analogy ig?

- Organizing books by a reference book
- Pick a book, place all “earlier” books on left, “later” on right
- Reference book is now correctly positioned
- Repeat for each pile

Quick Sort: Partition Algorithm

Partition Goal: Rearrange array so that:

- Elements $<$ pivot are on left
- Elements $>$ pivot are on right
- Pivot is in correct final position

Lomuto Partition Scheme (using last element as pivot)

1. Choose pivot: `pivot = arr[high]`
2. Maintain pointer `i` for "boundary" of smaller elements
3. For each element `j` from `low` to `high-1`:
   - If `arr[j] < pivot`: swap with position `i`, increment `i`
4. Finally: swap pivot with position `i+1`
5. Return pivot position

Quick Sort: Visualization

Quick Sort Viz

Quick Sort: Algorithm

Time Complexity

- Best case: Pivot divides array evenly

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n) = O(n \log n)$$

- Worst case: Pivot is smallest/largest (already sorted!)

$$T(n) = T(n-1) + O(n) = O(n^2)$$

- Average case: $O(n \log n)$

Space complexity: $O(\log n)$ for recursion stack

Quick Sort: Pivot Selection

The Problem

- Bad pivot choices -> worst case $O(n^2)$
- Already sorted array is worst case for "last element" strategy!

Strategies

1. Random pivot: Choose random element (probabilistic guarantee)
2. Median-of-three: Pick median of first, middle, last
3. IntroSort: Switch to Heap Sort if recursion too deep

In practice

- With good pivot selection, Quick Sort is often fastest comparison sort
- Cache-friendly: Works on contiguous memory

Quick Sort: Pros and Cons

Advantages

1. Fast in practice: Cache-friendly, small constants
2. In-place: Only $O(\log n)$ space for recursion
3. Average case: $O(n \log n)$ with good pivot selection

Disadvantages

- Worst case: $O(n^2)$ if bad pivots (rare with good strategy)
- Not stable: Equal elements might be reordered
- Fragile: Easy to implement incorrectly (e.g., overflow in midpoint)

When to use?

- Default choice for general-purpose sorting
- When memory is constrained

Practice Problems

Problem 1: Distinct Numbers

Problem Statement Given n numbers, count how many distinct numbers are present.

Link: https://cses.fi/problemset/task/1621

Intuition

- If array is sorted, equal numbers are adjacent
- Just count how many times the value changes!

Solution

1. Sort the array: $O(n \log n)$
2. Scan through, count when `arr[i] != arr[i-1]`

Time: $O(n \log n)$ | Space: $O(1)$

Problem 2: Sort Colors

Problem Statement Sort an array containing only 0s, 1s, and 2s in a single pass.

Link: https://leetcode.com/problems/sort-colors/

Intuition

- This is 3-way partitioning (used in Quick Sort optimization!)
- Maintain three regions:
  - [0...low-1]: all 0s
  - [low...mid-1]: all 1s
  - [high+1...n-1]: all 2s
  - [mid...high]: unknown/unprocessed

Solution:

1. Initialize: low = 0, mid = 0, high = n-1
2. While mid <= high:
   - If arr[mid] == 0: swap with low, low++, mid++
   - If arr[mid] == 1: mid++
   - If arr[mid] == 2: swap with high, high--

Time: $O(n)$ | Space: $O(1)$

Problem 3: Sum of Two Values

Problem Statement Given n numbers and target x, find two numbers whose sum is x.

Link: https://cses.fi/problemset/task/1640

Brute Force Intuition

- Try all pairs: $O(n^2)$
- Too slow for $n = 2 \times 10^5$!

Insight

- If array is sorted, we can use two pointers!
- Start with smallest and largest
- If sum too small: move left pointer right (need larger sum)
- If sum too large: move right pointer left (need smaller sum)

Solution

1. Store (value, original_index) pairs
2. Sort by value
3. Two pointers: left = 0, right = n-1
4. Move pointers based on sum comparison

Time: $O(n \log n)$ for sorting + $O(n)$ for two pointers = $O(n \log n)$

Space: $O(n)$ for storing pairs

Comparison Table

Algorithm	Time (Avg)	Time (Worst)	Space	Stable
Bubble	$O(n^2)$	$O(n^2)$	$O(1)$	Yes
Selection	$O(n^2)$	$O(n^2)$	$O(1)$	No
Insertion	$O(n^2)$	$O(n^2)$	$O(1)$	Yes
Merge	$O(n \log n)$	$O(n \log n)$	$O(n)$	Yes
Quick	$O(n \log n)$	$O(n^2)$	$O(\log n)$	No

Takeaway Problems!

Ferris Wheel

There are n children with given weights. Each gondola holds 1-2 children with total weight `<= x`. Find minimum gondolas needed.

Movie Festival

n movies with start and end times. Maximum number of movies you can watch entirely?

Questions?

Thank you!